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CALCULATION OF IRRADIATION COEFFICIENTS FOR A TRUNCATED CIRCULAR CONE

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Formulas for calculating the coefficient of angular irradiation of the side surface of a truncated circular cone by its base and the coefficients of mutual angular irradiation of regions of this surface have been derived.

The temperature regimes of heat exchangers should be calculated with account for the heat emitted by their heat-exchange surfaces. This is of particular importance for exchangers operating under high-temperature conditions and exchangers having surfaces with an inhomogeneous temperature distribution. For the purpose of intensification of the heat exchange in radiative slot recuperators, their surfaces are made in the form of a cone in a number of cases [1]. In the case where the temperature distribution over the inner surface of a truncated circular cone is inhomogeneous, the following coefficients of irradiation should be used in calculations: φ_{12}, coefficient of angular irradiation of the upper base by the lower base, i.e., of the circle of radius R_2 located at a distance H from the base by the circle of radius R_1; ψ_{1x}, coefficient of angular irradiation of a part of the side surface, occupying the region from the base to the height x, by the lower base; $\varphi_{1,dx}$, coefficient of angular irradiation of a differentially small strip of the side conical surface, located at a height x from the base, by the lower base; $\varphi_{d\xi,dx}$, coefficient of angular irradiation of a strip located at a height x by a strip located at a height ξ. Only the expressions for the coefficient φ_{12} are presented in reference books [2–4]. Expressions for the other coefficients can be obtained in the following way. Let us rewrite formula (22) from Table 15.4 presented in [4]:

$$\varphi_{12} = \frac{1}{2}\left[X - \sqrt{X^2 - 4\,(D_2/D_1)^2}\right],$$

where $X = 1 + \dfrac{4h^2 + D_2^2}{D_2^2}$, D_1 and D_2 are the diameters of the upper and lower bases of the truncated circular cone, and h is the height of this cone, in the form

$$\varphi_{12} = 0.5\,\left(1 + A^2 + B^2 - \sqrt{1 + (A^2 + B^2)^2 + 2\,(A^2 - B^2)}\right). \tag{1}$$

Here $A = (\xi - x)/(R_1 - x \cot \alpha)$ and $B = (R_1 - \xi \cot \alpha)/(R_1 - x \cot \alpha)$. Formula (1) determines the coefficient of irradiation of the circle located at a height ξ (circle ξ) by the circle located at a height x (circle x). The coefficient φ_{12} is obtained at $x = 0$ and $\xi = H$.

Using the closure property of the irradiation coefficients [4], we will determine the coefficient of irradiation of the side surface, located between the circles x and ξ, by the circle x:

$$\varphi_{x\xi} \equiv \varphi = 1 - \varphi_{12} = 0.5\,\left(1 - A^2 - B^2 + \sqrt{1 + (A^2 + B^2)^2 + 2\,(A^2 - B^2)}\right). \tag{2}$$

The coefficient φ_{1x} is obtained when $x = 0$ and $\xi = H$. The coefficient of irradiation of a differentially small conical strip of the side surface, located at a height ξ, by the circle x can be determined as

$$\varphi_{x,d\xi} = \varphi'_{\xi} d\xi = \frac{((A^2 + B^2)\,(AA'_{\xi} + BB'_{\xi}) + AA'_{\xi} - BB'_{\xi})}{\sqrt{1 + (A^2 + B^2)^2 + 2\,(A^2 - B^2)}}\, d\xi - (AA'_{\xi} + BB'_{\xi})\, d\xi\,, \tag{3}$$

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where $A'_\xi = 1/(R_1 - x \cot \alpha)$ and $B'_\xi = -\cot \alpha A'_\xi$.

At $x = 0$, replacing ξ by x, we obtain

$$\varphi_{1,dx} = \left(\frac{\dfrac{x^2}{\sin^2 \alpha} - 3R_1 x \cot \alpha + 2R_1^2}{\sqrt{(x/\sin \alpha)^2 - 4R_1 x \cot \alpha + 4R_1^2}} + R_1 \cos \alpha - \frac{x}{\sin \alpha} \right) \frac{dx}{R_1^2 \sin \alpha} . \quad (4)$$

Using the reciprocity property of the irradiation coefficients [4], we represent $\varphi_{d\xi,dx}$ in the form

$$\varphi_{d\xi,dx} = 0.5\, (\varphi''_{\xi x}\, (R_1 - x \cot \alpha) - 2\varphi'_\xi \cot \alpha) \sin \alpha \frac{dx}{B} , \quad (5)$$

where

$$\varphi''_{\xi x} = \frac{2\,(AA'_x + BB'_x)\,(AA'_\xi + BB'_\xi) + (A^2 + B^2)\,(A'_x A'_\xi + B'_x B'_\xi + AA''_{\xi x} + BB''_{\xi x}) + (A'_x A'_\xi - B'_x B'_\xi + AA''_{\xi x} - BB''_{\xi x})}{\sqrt{1 + (A^2 + B^2)^2 + 2\,(A^2 - B^2)}} -$$

$$- \frac{2\,((A^2 + B^2)\,(AA'_\xi + BB'_\xi) + AA'_\xi - BB'_\xi)\,((A^2 + B^2)\,(AA'_x + BB'_x) + AA'_x - BB'_x)}{\sqrt{(1 + (A^2 + B^2)^2 + 2\,(A^2 - B^2))^3}} -$$

$$- (A'_x A'_\xi + B'_x B'_\xi + AA''_{\xi x} + B''_{\xi x}) ;$$

$$A'_x = \frac{\xi \cot \alpha - R_1}{(R_1 - x \cot \alpha)^2} ; \quad B'_x = -\cot \alpha A'_x ;$$

$$A''_{\xi x} = \frac{\cot \alpha}{(R_1 - x \cot \alpha)^2} ; \quad B''_{\xi x} = -\cot \alpha A''_{\xi x} .$$

Introducing a local coordinate system with an origin at a height x ($x = 0$) and denoting the difference $(\xi - x)$ by x, we will obtain the following expression for $\varphi''_{\xi x}$:

$$\varphi''_{\xi x} = \left[\frac{4x^2 \cos \alpha}{R^4 \sin^4 \alpha} - \frac{3x}{R^3 \sin \alpha} \left(\frac{3 \cos^2 \alpha}{\sin \alpha} + \frac{1}{\sin^3 \alpha} \right) + \frac{\cos \alpha}{R^2} \left(\frac{6}{\sin^2 \alpha} + 2 \cot^2 \alpha + 2 \right) + \frac{\sin \alpha}{Rx} \right] \times$$

$$\times \left[\left(\frac{x}{R \sin \alpha} \right)^2 - \frac{4x \cot \alpha}{R} + 2 \right]^{-1/2} - 2 \left[\frac{x^2}{R^3 \sin^3 \alpha} - \frac{3x \cos \alpha}{R^2 \sin^2 \alpha} + 2\, \frac{\sin \alpha + \cos \alpha}{R} \right] \times$$

$$\times \left[\frac{x^2 \cos \alpha}{R^3 \sin^3 \alpha} - \frac{x}{R^2 \sin^2 \alpha} (1 + 3 \cos^2 \alpha) + \frac{1}{R} (\cot \alpha\, (4 + \cos^2 \alpha) + \sin \alpha \cos \alpha) - \frac{1 + \sin^2 \alpha}{x} \right] \times$$

$$\times \left[\left(\frac{x}{R \sin \alpha} \right)^2 - \frac{4x \cot \alpha}{R} + 2 \right]^{-3/2} - \frac{2x \cos \alpha}{R^3 \sin^3 \alpha} + \frac{1 + \cos^2 \alpha}{R^2 \sin^2 \alpha} , \quad (6)$$

where R is the radius of the circular cross section of the cone, located at a height x.

In the particular case where $\alpha = \pi/2$, from (4) we will obtain formula (11) from Table 15.4 of [4] for the coefficient of irradiation of a strip of the side surface of a circular cylinder by its base, and from (5) we will obtain formula (3) from the indicated table for the coefficient of mutual irradiation of cylindrical strips.

Thus, the dependences considered in the present work generalize formulas presented in reference works for calculating the coefficients of irradiation of elements of heat-exchange surfaces shaped as a truncated circular cone.

NOTATION

H, height of a truncated circular cone, m; R_1, R_2, radii of the lower and upper bases of the cone respectively, m; x, ξ, coordinates of strips of the side surface measured along the axis of the cone from its lower base, m; α, angle of inclination of the generatrix of the cone to the base plane; φ, coefficient of angular irradiation. Subscripts: 1 and 2, lower and upper bases of the cone.

REFERENCES

1. E. L. Mediokritskii and V. E. Loginov, Configuration of the heat-exchange surface of radiative slot recuperators, *Izv. Vyssh. Uchebn. Zaved.*, *Chern. Metallurgiya*, No. 11, 139–140 (1982).
2. A. G. Blokh, Yu. A. Zhuravlev, and L. N. Ryzhkov, *Radiative Heat Transfer. Handbook* [in Russian], Énergoatomizdat, Moscow (1991).
3. A. D. Klyuchnikov and G. P. Ivantsov, *Radiative Heat Transfer in Fire-Engineering Devices* (*Engineering Solution of Problems*) [in Russian], Énergiya, Moscow (1970).
4. S. S. Kutateladze, *Heat Transfer and Hydrodynamic Resistance* [in Russian], Énergoatomizdat, Moscow (1990).